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# Time Dependent Production Inventory Model with Random Switching Time for Non-Perishable Items

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## Abstract

*This paper analyzes a single product stochastic inventory model with two different rates of production, where demand follows Poisson distribution. It is assumed that products have infinite life-time that is not decay for the time under consideration. It is assumed that when the system reaches on a predetermined level, the system is converted to ON mode from OFF mode with a significant switching time with exponential parameter* $\tau$. *During switching time, no demand will be met. And hence, the demand during switching time is lost forever. Here backlogs are allowed and during backlogs, Production rate is higher than that of normal production time. Some system characteristics are displayed with time variation and total cost functions are derived. Finally, graphical representations are made.*

**Keywords:** Poisson distribution, Product Life-Time, Switching Time, Backlogs, Level Dependent
**JEL Classification**: D25, E23, L23, M11

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## 1.0 Introduction

Inventory control is one the most critical and important responsibilities for operation managers. Managing inventory control becomes difficult in stochastic situations. It is assumed here that products have infinite shelf time that does not decay for the time under consideration, even though in a number of practical situations, a certain amount of decay or waste is experienced on the stocked items. In the most inventory models, researchers basically make steady state analysis rather than

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transient. In order to understand the model clearly both the types of analysis are required. Transient analyses are basically time dependent. Sometimes, researchers need to know the data on the conduct of the framework up to time, when the framework has not yet achieved unfaltering state conditions. Transient examination is fitting in applications, for example, producing frameworks, frameworks with disappointment states, weak queueing frameworks, and frameworks with fluctuating or non- stationary outstanding burdens. Transient investigation is vital for considering queueing measures, for example, execution over limited intervals, affectability examination, and first section time calculation, settling time calculation, and determining the conduct of models as they approach harmony.

## 2.0 Literature Review

Transient analysis is basically time dependent. Researchers are very often interested in the system behavior with respect to time for taking tactical decision. Hence the analysis has some importance. Vey less amount of work has been published based on transient analysis than the steady state case. Elhafsi and Bari (1996) made Transient and steady-state analysis of a manufacturing system with setup changes. They dealt with the optimal scheduling of a one-machine two-product manufacturing system with setup, operating in a continuous time dynamic environment. For the steady-state, the optimal cyclic schedule (Limit Cycle) is determined. To solve the transient case, the system's state space is partitioned into mutually exclusive regions such that each region is associated with an optimal control policy. A novel algorithm (Direction Sweeping Algorithm) is developed to obtain the optimal policy that minimizes the sum of inventory and backlog costs. Krishnamoorthy and Islam (2004), discussed (s, S) inventory system with postponed demands. They assume that customers arrive to the system according to a Poisson process with rate $\lambda>0$ When inventory level depletes due to demands or service to a pooled customer, an order for replenishment is placed. The lead time is exponentially distributed with parameter $\gamma$. When inventory level reaches zero, the incoming customers are sent to a pool of capacity M. Any demand that takes place when the pool is full and inventory level is zero, is assumed to be lost. After replenishment, as long as the inventory level is greater than s, the pooled customers are selected according to an exponentially distributed time lag, with rate depending on the number in the pool. They made steady state and transient analysis for the system performances.

Al Hanbali and Boxma (2009) studied the transient behavior of a state dependent M/M/1/K queue during the busy period. They derived in closed-form the joint transform of the length of the busy period, the number of customers incurred losses during the busy period. For two special cases called the threshold policy and the static policy they determined simple expressions for their joint transform. Ammar (2014) obtained the time-dependent solution of a two- heterogeneous servers queue with impatient customers using probability-generating function. Islam et.al. (2020) analyzed a time dependent (s, S) Inventory system in supply chain environment. They considered a level dependent perishable inventory system, where raw materials arrive from two warehouses which are situated nearby the central processing unit. Arrival of demands follows Poisson process with rate. Production takes place when at least one component of each category is available in both the warehouses. Replenishment for the warehouses occurs in negligible time once the component amount reaches to zero unit. It is assumed that the initial inventory level is in S and system is in OFF mode. Inventory level decreases due to demands and perishability. When the inventory level reaches ***s*** then the system converted to ON mode from OFF mode. The production follows exponentially distributed with parameterμ. Perishability follows exponentially distributed with parameter θ. Perishability will be level dependent that is rate of perish ability will depend on the amount of inventory available in the stock. Transient State analysis is made and some system characteristics are evaluated by numerical illustration.

A Markovian queuing system with multiple-heterogeneous servers, reneging and retention of reneging customers is studied by Kumar and Sharma (2021). In their study, the transient analysis of the model is performed using probability generating function technique. Important measures of performance like expected system size, average reneging rate and average retention rate are presented. The transient behavior of state probabilities is studied numerically. The transient behavior of performance measures is also studied. Further, a comparative analysis is performed which shows that the multiple-heterogeneous servers queuing model performs better than the multiple-homogeneous servers queuing model. Finally, the steady-state solution of the model is also obtained. This work is the extension of the works of Mohammad Ekramol Islam et al. (2019). In that paper, inventory has a finite life-time that is perishable in nature but in the present paper inventory has a property of infinite shelf-life.

## 3.0 Assumptions and Notations

### a) Assumptions

i. Two production rates are considered where production rate is higher during backlogs than normal time. The logic behind this is backlog demand is known and need to serve that first.

ii. When inventory level reaches at a predetermine level –N, the system converted to ON mode from OFF mode with parameter $\tau$ which follows exponential distribution.
iii. Inventory has a finite life-time or non-perishability in nature.
iv. During switching time, no demand is allowed, so demand during switching time is lost forever.
v. If inventory level reaches order level, production process will be switched off.

**b) Notations**

i. $\lambda$ →Arrival rate,
ii. $\mu$ → Production rate during backlogs,
iii. $\delta$ →Normal production rate,
iv. S →Maximum inventory level,
v. $\tau$ →Switching time
vi. $I(t)$ Inventory level at time t,
vii. $E \rightarrow E_1 \cup E_2$ is the state space of the process

where, $E_1 = \{(t,0): t = -N+1, \ldots, S\}$ and
$E_2 = \{(t,1): t = -N+1, \ldots, S-1\}$

## 4.0 Model Analysis

We assume that the inventory level S at the starting point and the system is in OFF mode. Arrival of the demand follows Poisson distribution with rate $\lambda$. Inventory level will deplete due to customers' demand. When inventory level reaches the level –N then the system will be switched ON mode. In the inventory system, inventory level $I(t)$ takes the value

$A = \{-N, -N+1, \ldots, 0, 1, 2, \ldots, S\}$

To get a two dimensional Markov process we introduce the $\{X(t), t \geq 0\}$

Where, *X(t)* is defined by

$$X(t) = \begin{cases} 1 & \text{when production is ON mode} \\ 0 & \text{when production is OFF mode} \end{cases}$$

Now, the infinitesimal generator of the two dimensional Markov process$\{I(t), X(t); t \geq 0\}$ is defined on the state space E. it is noted that the Markov process is a pure birth and death process during the transition from the state $(S,0)$ through the state $(S-1,0), \ldots, (-N+1,0)$ when the production process is in OFF mode. When inventory level in the state $(-N,0)$ then the system is switched ON mode from off mode. Switching time follows exponential distribution with parameter $\tau$ and reached the state $(N,1)$from $(-N,0)$. From this state onward the process will be pure birth and death process until it reaches the level $(S,0)$.

Let us assume $I(0)=S$ and $X(0)=0$. Let us consider the transition probabilities:
$P_{(S,0)\,(i,j)}(t) = P\{I(t),X(t) = (i,j)|I(0),X(0) = (S,0)\}$

From now onwards we can write

$$P_{(i,j)}(t) \text{for} P_{(S,0)\,(i,j)}(t)$$

Kolmogorov difference differential equations for the system $P_{(i,j)}(t)$ are given bellow:

When system is OFF mode:

$$P'_{(S,0)}(t) = -\lambda P_{(S,0)}(t) + \delta P_{(S-1,1)}(t) \quad [1]$$

$$P'_{(i,0)}(t) = -\lambda P_{(i,0)}(t) + \lambda P_{(i+1,0)}(t);\ i = S-1, \dots, 0 \quad [2]$$

$$P'_{(i,0)}(t) = -\lambda P_{(i,0)}(t) + \lambda P_{(i+1,0)}(t)\,;\ i = -1,\ \dots\ , -N+1 \quad [3]$$

$$P'_{(-N,0)}(t) = -\tau P_{(-N,0)}(t) + \lambda P_{(-N+1,0)}(t) \quad [4]$$

When the system is ON mode:

$$P'_{(S-1,1)}(t) = -(\lambda+\delta)P_{(S-1,1)}(t) + \delta P_{(S-2,1)}(t) \quad [5]$$

$$P'_{(i,1)}(t) = -(\lambda+\delta)P_{(i,1)}(t) + \lambda P_{(i+1,0)}(t) + \delta P_{(i-1,1)}(t);\ i=S-2, \dots, 0 \quad [6]$$

$$P'_{(i,1)}(t) = -(\lambda+\delta)P_{(i,1)}(t) + \lambda P_{(i+1,1)}(t);\quad i=-N+1, \dots,-2,-1 \quad [7]$$

$$P'_{(-N,1)}(t) = -(\lambda+\delta)P_{(-N,1)}(t) + \tau P_{(-N,0)}(t) + \lambda P_{(-N+1,1)}(t) \quad [8]$$

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We solve this system of ordinary differential equations by using the Runge-Kutta method of fourth order based on specific parameters. We have plotted the graphs on the basis of ODE's and performance measures. The effect of various parameters on the system performance measures such as expected number of customers in the system and mean waiting time in the system are studied. MATLAB software is used to develop the computational program.

## 5.0 Some Performance Measures

**a) Mean inventory level in the system**

Let the expected inventory level

$$Ls(t) = \sum_{i=1}^{S} iP_{(i,0)}(t) + \sum_{i=1}^{S-1} iP_{(i,1)}(t)$$

**b) Expected backlogs in the system**

Let the expected backlogs

$$Lb(t) = \sum_{i=-N}^{-1} |i|P_{(i,0)}(t) + \sum_{i=-N}^{-1} |i|P_{(i,1)}(t)$$

**c) Expected number of customer lost**

Let expected number of customer lost

$$CL(t) = \lambda P_{(-N,o)}(t) + \lambda P_{(-N,1)}(t) + \tau P_{(-N,0)}(t)$$

**d) Expected switching time $ST(t) = \tau$.**

**e) Expected total cost of the system**

$$ETC(t)) = L + c_1 Ls(t) + c_2 Lb(t) + c_3 CL(t) + c_4 ST(t).$$

## 6.0 Numerical Results and Discussion

In all numerical computations, the model parameters are taken as
$S$=3, $N$=2, $\lambda$=1, $\tau = 0.21$, $\mu = 3$, $\delta = 2$, L=25,
$c_1 = 0.15, c_2 = 0.25, c_3 = 0.35. c_4 = 1.$

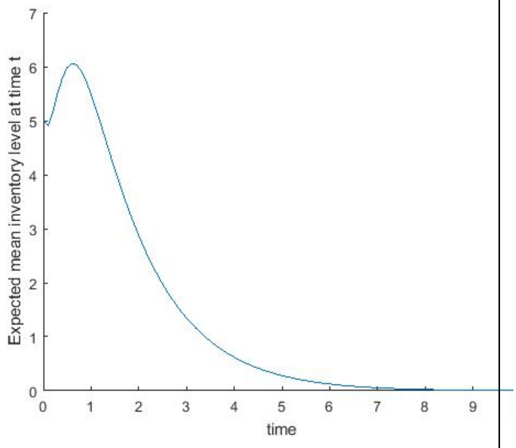


Fig.1: Mean inventory level vs Time

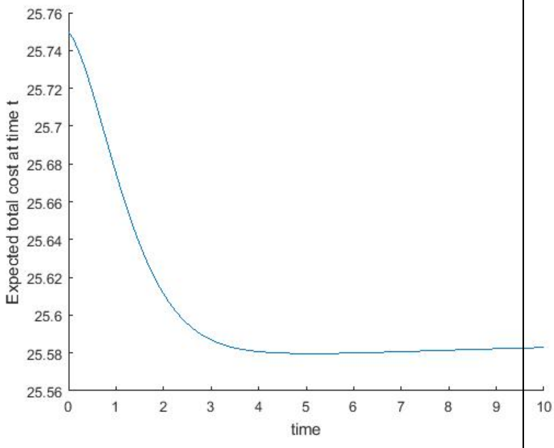


Fig.2: Expected total cost vs Time

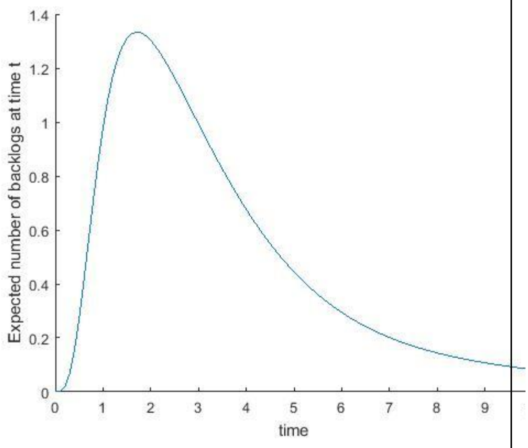


Fig.3: Expected backlogs vs Time

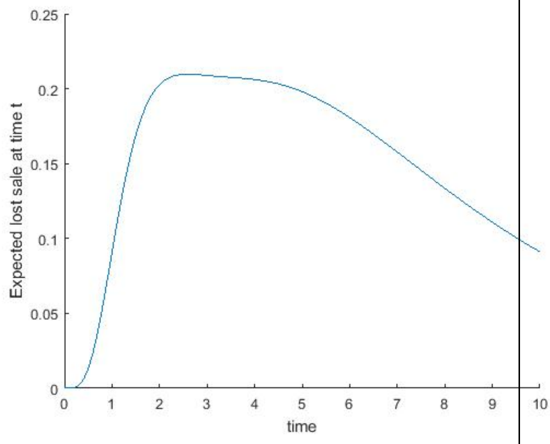


Fig.4: Expected lost sale vs Time

Numerical outcomes have been gotten by applying Runge-kutta fourth request technique in the arrangement of standard differential conditions [1] through [8] with the assistance of computational programming MATLAB R2016a for the parametric qualities steady with time interim $0 \leq t \leq 10$ as appeared in the diagrams. Diagrams of different parameters versus time have been demonstrated the fig.1 through fig.4.

Fig.1 investigates that the stock raised quickly for short time but for long time interim it decrease at lower rate. Fig.2 exhibits the normal aggregate expense of the framework, which demonstrate that at first aggregate expense, is higher than whatever remains of era considered in the model. Fig.3 demonstrates the average number of backlogs increment at the beginning stage however after some time, at a certain rate it decreases in the end. From Fig.4, we see that the rate of lost deal is very high at the beginning time after some time, which decreases as the production rate is higher than the normal production period.

## 7.0 Conclusion

Examination of single product model under time subordinate section and organization has been made. Under the examination the numerical results for various execution measures have been procured by using Runge-kutta fourth demand system with MATLAB R2016a. Our model can be considered under different servers' course of action, which may give logically extensive game plan under time subordinate conditions to make the model progressively sensible.

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## Appendix
## MATLAB R2016a code
## .m file

```
function [dy_dt] = p04(t, D)
 % IV,  I,  IVsol – Independent variables
 % DV,  D,  DVsol – Dependent variables
 % y( i, 2) off mode
 % y(i, 1) on mode

  y(62) = D(1); y(52) = D(2); y(42) = D(3); y(32) = D(4); y(22) = D(5);
  y(12) = D(6); y(61) = D(7); y(51) = D(8); y(41) = D(9); y(31) = D(10);
  y(21) = D(11); y(11) = D(12);


 dy_dt = [ -4*y(62)+2*y(51); -3*y(52)+4*y(62); -2*y(42)+3*y(52);
-y(32)+2*y(42);   -y(22)+y(32);    -0.21*y(12)+0.21*y(22);
-4*y(61)+2*y(52);  -5*y(51)+3*y(61); -4*y(41)+3*y(52)+2*y(51);
-4*y(31)+3*y(42)+2*y(41); -4*y(21)+4*y(31); -2*y(11)+0.21*y(12)+y(21);];

End
```

## Scriptfile.m

```
clear all; close all; clc

t = 0:0.1:10;
ic1=1;ic2=0;ic3=0;ic4=0;ic5=0;ic6=0;ic7=0;ic8=0;ic9=0;ic10=0;ic11=0;ic12=0;
y0 = [ ic1,ic2,ic3,ic4,ic5,ic6,ic7,ic8,ic9,ic10,ic11,ic12];
[t, Dv] = ode45('TA5', t, y0 );
```

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```
A=(Dv);

MIL=(Dv(:,11)+Dv(:,5))+2*(Dv(:,4)+Dv(:,10))+3*(Dv(:,3)+Dv(:,9))+4*(Dv(:,2)+Dv(:,8))+5*(Dv
(:,1)+Dv(:,7));
ENBL=((Dv(:,12)+Dv(:,6))+2*(Dv(:,5)+Dv(:,11)));
ENCL=((Dv(:,12)+Dv(:,6))+0.21*(Dv(:,6)));
L=25;

ETC=0.15*MIL+0.1*ENBL+0.35*ENCL+L;
hold on
disp([t, Dv])

plot(t, MIL ),xlabel('time'),ylabel('Expected mean inventory level at time t')
plot(t, ENBL),xlabel('time'),ylabel('Expected number of backlogs at time t')
plot(t, ENCL),xlabel('time'),ylabel('Expected lost sale at time t')
plot(t, ETC ),xlabel('time'),ylabel('Expected total cost of the system at time t')
```